CHAPTER 5

MULTI-PERIOD ECONOMIC DECISION CRITERIA

I. Requirement to Discount

This chapter presents methodology with which to make the comparison of investment or regulatory alternatives required by step 6 of the economic analysis process. The methodology accounts for the characteristic that benefits and costs occur over a number of years. It explicitly recognizes that otherwise equal benefits or costs which occur at different points in time will not be equal when viewed from a common point in time. Generally, a benefit will be worth more the sooner it is received, and a cost will be less the longer it is deferred. This economic phenomena is the result of two factors: the productivity of capital and the time preference of economic decision makers.

An observed characteristic of economic life is that production processes which employ capital--buildings, machines, organized methods such as assembly lines, etc.--are frequently more productive than other production methods. Such methods are not only able to recover the costs of the resources to build the capital, but return something in addition to this. This additional return, known as the net productivity of capital, provides an incentive to undertake every activity for which it exists. Unfortunately, there are insufficient resources to carry out all such projects.

At any particular time, the quantity of resources in an economic system is fixed. As a consequence, only some of the many activities capable of returning more than their cost can be undertaken. Rational decision making requires that those activities with greater returns over cost be undertaken before those with smaller returns until all investable resources are exhausted. The last activity undertaken before exhausting the investable resources should have a return less than or equal to all activities actually undertaken and greater than or equal to all activities not undertaken. This level of return, commonly expressed as an annual rate, is known as the marginal rate of return to capital. It represents the prevailing level of capital productivity that can be achieved at any particular time by investing resources.

Limited resources may be divided between current consumption and capital investment, which implies future consumption. However, there is a general predisposition for people to prefer current consumption over future consumption, or to have a positive time preference. In very poor subsistence level economic systems, immediate consumption of everything, or almost everything, may be necessary for survival. But even in wealthier systems, either because of general impatience, or the ever present probability of death, or some other reason, people are willing to forgo current consumption to invest in the future

only to a limited degree. The extent to which a person discounts future consumption is referred to as the rate of time preference. The rate at which society discounts future consumption is known as the social rate of discount.

Any investment requires that resources either be diverted from another investment or obtained by deferment of consumption. This gives rise to a cost, either in terms of the return that could be earned on capital in its next best alternative use--the marginal rate of return to capital--or the compensation that must be paid to induce people to defer an additional increment of current consumption--equivalent to their marginal rate of time preference. This cost is captured in benefit-cost analysis through discounting. Before proceeding, it is emphasized that the requirement to discount does not depend upon the existence of inflation. Rather it arises from the productivity of capital, peoples preferences for current over future consumption, and the scarcity of investable resources. Even in an inflationless world, discounting is required. (The appropriate treatment for inflation in investment analysis will be discussed in Chapter 7.)

II. Discounting Methodology

A. Mechanics

For a period of one year, an investment can be expected to grow at some rate, as shown by equation (5-1):

$$O_1 = I + rI = I(1+r) \tag{5-1}$$

where: I = the investment's initial value,

O_1 = the investment's value in one year, and

r = the growth rate.

For a period of two years, investment growth is given by:

$$O_2 = I(1+r)(1+r) = (1+r)^2 \tag{5-2}$$

Here the growth rate is applied, in succession, twice because the investment is allowed to grow for two years. Similarly, for a period of n years the growth rate is applied n times:

$$O_n = I(1+r)^n \tag{5-3}$$

The significance of equation (5-3) is that it indicates the extent to which resources invested today (*I*) can be transformed into outputs in the future (O_n) any growth rate *r*. In many instances, the amount that will be received in the future is known. The value today of an amount to be received in the future can be determined by dividing (5-3) through by $(1+r)^n$ to yield (5-4). It is equation (5-4) that is relevant to discounting.

$$I = \frac{O_n}{(1+r)^n} \tag{5-4}$$

It indicates that *I* is the present value of O_n after being discounted over *n* years at rate *r*.

Equation (5-4) can be extended to situations where outputs are generated and resources consumed in more than one period. This requires that several equations--one for each year--of the form of (5-4) be added together, as in (5-5):

$$I^1 + I^2 + \ldots I^n = \frac{O_1}{(1+r)^1} + \frac{O_2}{(1+r)^2} + \ldots + \frac{O_n}{(1+r)^n} \tag{5-5}$$

where: I^i = the initial investment associated with outputs in year *i*.

By defining O_t as the difference between benefits (B_t) and costs (C_t) in year *t* and their discounted value as their net present value, equation (5-5) may be rewritten in its usual form:

$$NPV = \sum_{t=0}^{k} \frac{(B-C)_t}{(1+r)^t} = \sum_{t=0}^{k} \frac{B_t}{(1+r)^t} - \sum_{t=0}^{k} \frac{C_t}{(1+r)^t} \tag{5-6}$$

where: *NPV* = the discounted net present value of a series of outputs and resource inputs, and

k = the total number of periods in the evaluation period of the project or regulation.

B. Discount Rate

As noted above, the discount rate represents the cost associated with diverting investment resources from alternative investments or from consumption. There is no general consensus, either conceptually or empirically, as to what this rate should be. A principal reason for this lack of consensus is that the rate of time preference of those who postpone consumption--savers--is typically significantly lower than estimates of the marginal rate of return to capital. This occurs in large part because of the impact of corporate and

personal income taxes. The marginal rate of return to capital is appropriately measured on a before tax basis whereas returns actually received by savers are net of taxes. Because people adjust their saving until they are indifferent between a dollar of current consumption and a dollar of future consumption plus the return they actually receive after taxation for deferring consumption, their time preference is equal to the after tax return to saving.

Four conceptual alternatives for the discount rate have been proposed. These are:

- the marginal social discount rate
- the marginal opportunity cost of capital
- a weighted average of the marginal social discount rate and the opportunity cost of capital
- the shadow price of capital

For most Federal investments and regulations, the Office of Management and Budget (OMB) requires use of the marginal cost of capital approach as a base case. OMB further recognizes the merit of the marginal social discount rate approach through a requirement to conduct sensitivity analysis with respect to the discount rate. The shadow price of capital approach is also permitted with prior OMB concurrence.

1. Marginal Social Rate of Discount

This rate indicates the compensation required by society to substitute future consumption for current consumption. Discounting a stream of benefits and costs at this rate produces the present value of this stream as viewed by society. It is often measured empirically as the after tax, after inflation riskless rate of interest, such as could be earned on U.S. Treasury debt less taxes. There are fundamental conceptual problems with this approach, however.

First, use of the rate of time preference will represent the opportunity cost of investment resources only if these resources are in fact diverted from consumption. If they actually come from another source, such as other investments or foreign borrowing, other rate(s) representing the cost of these resources is relevant.

Second, an observed interest rate net of taxes will measure the willingness of people to substitute future for current consumption only if they in fact do make such substitutions in response to the prevailing rate of interest. But many observers believe that consumers do not vary consumption in response to interest rates over the range of rates which typically prevail. Thus, observed rates measure only the rent earned on those resources available for investment, not peoples' rate of time preference.

Third, observed rates reflect the preferences of individual members of society who have finite lives and are currently living. Because society presumably has an indefinite life, it may well discount future consumption by less than its individual members. The issue is one of intergenerational equity--do current members of society have any more rights to consume existing resources than future members? It is most relevant when evaluating potential actions with very long lives where a significant portion or all of the benefits will accrue to those yet to be born. Evaluation of environmental policies and regulations are examples.

2. Marginal Opportunity Cost of Capital

This approach presumes that resources used for the investment under evaluation will in fact be diverted from private investment. The appropriate opportunity cost under these circumstances is what actually will be foregone in the private sector. It is measured by the return on additional investment in the private sector of the economy before taxes and net of inflation.

3. Weighted Average of the Marginal Opportunity Cost of Capital and the Social Rate of Discount.

This approach recognizes that resources required to undertake investments may actually come from both other investments and consumption. Accordingly, a weighted average is constructed where the weights represent the respective shares coming from consumption and other investment. Because it is an average of two other methods, this approach avoids the problem that funds may come from more than one source; most other problems remain.

4. Shadow Price of Capital Approach

The shadow price of capital approach explicitly recognizes that the marginal rate of return and the rate of time preference typically have different values and provides a conceptual solution to the discounting problem which incorporates both rates. Proceeding from the premise that consumption is the purpose of all economic activity, it expresses all benefit and cost flows of the decision under evaluation in terms of their consumption equivalents in the year in which they occur and then discounts them back to the present at the rate of time preference.

For benefits (which by definition are consumed in the current time period) and costs which affect outputs only in the current time period--operations costs for example--this is straight forward. These quantities are valued at their prevailing values in each time period. For capital investments which give rise to benefit flows in the future, the valuation process to express these amounts in current consumption equivalents is more involved. First, it is necessary to divide the amount invested in each period into that which is diverted from

current consumption and that which is obtained by forgoing other investments. Because that which is diverted from consumption represents foregone consumption, it is already valued in consumption units and its value can be used directly in the computation of present value. The challenge comes in expressing in its current consumption equivalent value the investment which is diverted from some other competing investment use.

In its next best use investment resources can be expected to give rise to a future stream of outputs, some of which will be invested and some of which will be consumed. Those which are reinvested will in turn give rise to their own future stream of outputs, some of which will be in turn invested and some of which will be consumed. This process will continue into the future, perhaps indefinitely. Under a wide range of reasonable circumstances, it is possible to calculate the present value of the stream of future output available for consumption that the initial investment will make possible. The present value of the future consumption yielded by one dollar of capital is known as the shadow price of capital.

If this approach were used, equation (5-6) would be restated as:

$$NPV = \sum_{t=0}^{k} \frac{B_t - [(1-c)S + c]\, C_t}{(1+i)^t} \tag{5-7}$$

where: c = the fraction of project costs coming from saving,

i = the rate of time preference, and

S = the shadow price of capital.

The term S--the shadow price of capital--effectively adjusts that investment cost drawn from alternative investment projects to account for the higher marginal return to capital, and the ultimate reduction in future consumption, that must be forgone to undertake this project. Randolph Lyon has derived the following expression for S assuming that geometric depreciation adequately represents economic depreciation of the capital stock:[1]

$$S = \frac{r - sr}{i + d - sr} \tag{5-8}$$

where: r = the marginal return to capital before depreciation,

s = the rate of savings from the gross return,

i = the rate of time preference, and

[1] Randolph M. Lyon, "Federal Discount Rate Policy, the Shadow Price of Capital, and Challenges for Reforms," *Journal of Environmental Economics and Management*, 18, no. 2, part 2 (March 1990), S29-S50.

d = the rate of depreciation.

Use of this approach effectively solves the problem of which rate to use at the conceptual level. It is the analytically preferred means of capturing the effects of Governmental projects on resource allocation. Unfortunately, its actual application is far from straight forward. The difficulty is that *S* can vary significantly as its determinants are varied over their plausible ranges.

C. Executive Branch Discount Rate Policy

The OMB establishes discount rate policy for most Executive Branch evaluations of investment and regulatory decisions. Its policy is outlined in OMB Circular A-94, "Guidelines and Discount Rates for Benefit-Cost Analysis of Federal Programs," (Revised--October 29, 1992).[2] OMB permits analyses to be conducted in either nominal or current year dollars or in constant dollars of a particular year, although nominal and constant dollars cannot be mixed-up in the same analysis. There is a preference for the use of constant dollars unless most of the underlying values are initially available in nominal dollars. Although some conversion from nominal to constant or vice-versa may be necessary to get all values into one form or another, the choice of nominal or constant dollars should be made so as to minimize the conversions required. If the analysis is conducted in nominal dollars, the discount rate selected should be a nominal one; if the analysis is conducted in constant dollars, the discount rate should be a real one. Real rates can be approximated by subtracting expected inflation from a nominal rate.

OMB effectively divides potential Government investments and regulations into four categories: (1) public investment and regulatory analyses, (2) lease-purchase and asset sale analyses, (3) cost-effectiveness and internal Government investments, and (4) combined projects. Discounting concepts and rates vary across categories.

1. Public Investment and Regulatory Analyses

OMB specifies that a base case analysis be conducted for potential actions using a real discount rate of 7 percent. Circular A-94 states that "this rate approximates the marginal pretax rate of return on an average investment in the private sector...." Selection of this rate implies OMB acceptance of the marginal opportunity cost of capital approach to discounting discussed above. However, Circular A-94 also states that "public investments and regulations displace both private investment and consumption," thus acknowledging the marginal social discount rate approach.[3] Rather than suggesting a weighted discount rate approach, OMB recognizes that investment resources may come from multiple sources with different opportunity costs by requiring that sensitivity analyses be conducted

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[2] A summary of applicability of the Circular A-94 is contained in Appendix A.

[3] "OMB Circular A-94" (Revised--October 29, 1992) p. 9.

at discount rates both higher and lower than 7 percent. The Office of Aviation Policy and Plans recommends that sensitivities be carried out at both 10 percent and 4 percent.

Alternatively, OMB recognizes that "using the shadow price of capital to value benefits and costs is the analytically preferred means of capturing the effects of Government projects on resource allocation in the private sector." Because of the practical problems involved with using this approach, it may be used only with OMB concurrence.[4]

2. Lease-Purchase and Asset Sale Analysis

These types of analyses do not address diversion of resources to a Government investment project. (For lease-purchase the decision to undertake the investment has presumably already been made under the public investment approach noted above. For Asset Sales the investments have already been made and are owned by the Government.) Rather, they concern asset ownership. In the case of lease-purchase, the question is should a particular asset which will be used by the Government be owned by the Government or by the private sector. The issue is viewed strictly as one of minimizing the Government's cost. Accordingly, OMB requires that the stream of lease payments and the stream of ownership costs--purchase plus operation--each be discounted by the Treasury's borrowing rate on marketable securities of comparable maturity to the period of analysis, expressed in real or nominal terms, depending on how the cost streams are measured.[5, 6]

In the case of asset sales, the question is whether an asset owned by the Government should be sold to the private sector.[7] Gains in social efficiency may be possible under private ownership which subjects an asset to market discipline and private incentives. If so, presumably the private sector will be willing to pay more for it than it is worth to the Government. To determine if the asset is worth more under private sector ownership, it is necessary to determine its value under Government ownership and then ascertain if private parties are willing to pay more for it. Circular A-94 requires that the assets present value

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[4] "OMB Circular A-94" (Revised--October 29, 1992) p. 9.

[5] Real and nominal Treasury borrowing rates for alternative maturities are published by OMB in Appendix C to Circular A-94. This appendix is updated each February.

[6] For the reasons indicated above in Section II.B., it should be noted that the Treasury borrowing rate does not *conceptually* correspond to any of the four discount rate alternatives. It is strictly the government's cost of borrowing.

[7] Governmental assets which are typically candidates for divestiture to the public are those associated with the sale (as opposed to free provision) of goods and services to the public by the government, often in competition with the private sector. Examples would include Federal credit programs and the loan portfolios which they generate and the Postal Service. Governmental assets which are used to provide benefits to the public which generate little or no revenues are not candidates for sale in that they would attract no buyers.

be determined by discounting its future earnings stream--after adjustment for expected defaults or delays, Government administrative costs, and expected increases or decreases in asset value--using the Treasury borrowing rate for a term equivalent to the life of the asset, either real or nominal, depending on how the earnings are measured. Further, because the private sector will tend to discount earnings streams at a higher rate than the Government borrowing rate to account for risk, when there is evidence that Government assets can be used more effectively in the private sector, valuation analyses for these assets should include sensitivity comparisons that discount the returns from such assets with the rate of interest earned by assets of similar riskiness in the private sector.

3. Cost-Effectiveness Analyses and Internal Government Investment

Cost-effectiveness analyses are undertaken in a situation where the decision has been made to produce a particular output and it is necessary to determine the lowest cost method to produce it. Analyses of Government investments address the viability of proposed projects where the benefits take the form of increased Federal revenues or decreased Federal costs. For both these cases, Circular A-94 indicates that the comparable-maturity Treasury borrowing rate, either real or nominal depending on how benefits and costs are measured, be used as the discount rate. Specifying this rate will have the effect of selecting projects which minimize the present value of costs or maximize the present value of benefits to the Government.[8]

4. Combined Projects

Many investment projects yield both cost savings to the Government and external social benefits. Circular A-94 specifies that these be discounted at the base case rate of 7 percent, as required for public investment and regulatory analyses, unless it is possible to "*allocate* the investment's costs between provision of Federal cost savings and external social benefits." Where such an allocation is possible, the project can be treated as two separate projects with the part yielding benefits to the Federal Government discounted at the Treasury rate and the part yielding external social benefits discounted at the 7 percent real discount rate.[9]

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[8] It should be noted that these analyses, unlike lease-purchase decisions or asset sales, involve decisions as to the amount of investment resources to be diverted from the private sector and/or the time distribution of these investments. As such there is an opportunity cost of these resources. The OMB policy, in effect, permits the Federal Government to consider its borrowing cost as the opportunity cost of capital rather than the marginal pre-tax return to capital which is required for investments which provide benefits to the private sector. While this procedure will lead to projects that minimize costs to the Government, it may divert resources away from more productive uses in the private sector.

[9] "OMB Circular A-94" (Revised--October 29, 1992) p. 11.

Although many FAA investment projects yield both benefits to the Federal Government and external social benefits, the costs are frequently difficult to allocate between these two. The following guidance is offered:

- The total project should first be evaluated with the 7 percent base case real discount rate. If it has positive net present value, it is probably unnecessary to go further.

- Should a project not have a positive net present value at the 7 percent real rate, the following steps may be taken:

    * If Governmental cost savings are a large component of the benefits, evaluate the project at the Treasury borrowing rate considering only the Governmental benefits. (So doing will ensure that a relatively small amount of external social benefits associated with what is fundamentally a cost saving project would not *taint* the outcome of the analysis by requiring the higher 7 percent be used.)

    * Where there are significant benefits in both categories and a positive net present value does not result from the above procedures, costs may be allocated between the two categories. The amount of project cost allocated to Governmental cost saving should be at least as large as the costs that could be avoided if the project were redesigned so as not to yield Governmental benefits (avoidable costs of the Governmental cost savings) but no larger than the cost of producing the Governmental benefits independently of the external social benefits (stand alone costs). Should avoidable costs exceed stand alone costs, the allocation should be set at the level of stand alone costs. Within these two extremes, a reasonable allocation based on accepted cost accounting procedures may be employed. In addition, the allocation at which the project would have a zero net present value should be identified with sensitivity analysis.

D. Evaluation Period

The number of years over which the benefits and costs of an investment or regulation should be considered may be designated as the evaluation period. Three time periods are of concern in determining the evaluation period: requirement life, physical life, and economic life. The requirement life is that period over which the benefits of the good or service to be provided or mandated by regulation will be greater than the costs of producing it. It can be for a very short period of time such as a requirement to provide special air traffic control services to an air show held at an otherwise uncontrolled airport. Or it may be for a very long period of time such as the provision of en route surveillance coverage.

The physical life is that period for which facilities and equipment can be expected to last. It is to a considerable degree under the control of the decisionmaker. Not only can alternative facilities and equipment with different physical lives resulting from inherent quality differences be procured, but maintenance policies can be varied to alter an asset's physical life after it has been put in service.

The economic life is that period over which an asset can be expected to meet the requirements for which it was acquired at the lowest achievable cost. Thus, by definition, economic life is less than or equal to requirement life. Economic life may be equal to physical life but it is frequently less. If less, this indicates that it is not efficient to operate the asset as long as possible. Rather, it is cheaper to replace it. The need to replace often occurs as the consequence of ever rising maintenance costs, particularly for relatively old items. Estimates of economic lives should be based on actual information where possible. In the absence of such information, the guidelines in Table 5-4 may be used. These guidelines have been synthesized from a number of sources and are intended to represent actual practice with respect to broad classes of assets. As such they may be regarded as approximations to economic lives.

The evaluation period may be defined with respect to either the length of time over which the good or service to be produced will be required or the economic life of the investment required to produce it. Because either method will yield the same results, the choice is dependent on the circumstances of the analysis and can be made based on considerations of practicality. Although the evaluation period may be defined with respect to either requirement life or economic life, investment projects or regulations requiring specified investments--design regulations--are usually best evaluated over their economic lives.[10] Use of the requirement life method would require the assumption that the facilities and equipment would be replaced at the end of each economic life period forever. Such assumption, while not improper, would add little to the analysis. Moreover, it might obscure the fact that equipment performance is likely to improve with time and that better performance, lower cost replacements are likely to be available in the future.

[10] In practice, physical life is often used. To the extent this diverges from economic life, costs will be overstated which may result in otherwise potentially viable projects failing the benefit-cost test.

TABLE 5-1
ASSET LIFE GUIDELINES

Item:	Asset Life in Years
Aircraft:[a] [c] [d] [e] [f]	
Airframes	20-40
Avionics	10-20
Flight Instruments	10-20
Airports:[a] [g]	
Lighting Systems	15-25
Pavements	20-40
Terminals (Permanent Structures)	45+
Automation:[c] [d] [f]	
FAA Developed Hardware	15-20
FAA Developed Software	10-15
Mainframe Computers	10-15
High-End Work Stations	5-7
PC Workstations	5-7
PC Workstation Application Software	3
Communications:[c] [d] [f]	
Radios	8-15
Telephone/Telegraph/Teletype	10-20
Miscellaneous	10-20
Tower/TRACON Voice Switches	10
Enroute Voice Switches	10-15
Operating Equipment:[c] [d] [e]	10-20
Navigation/Landing Equipment:[c] [d]	20-25
Structures:[b] [c] [d] [f]	
Permanent	45+
Semi-Permanent	25-45
Temporary	15-25
Surveillance Radars:[a] [c] [d]	20-25
Weather:[c] [d]	
General-Purpose Weather Sensors	10-20
Weather Radars	20-25
Vehicles:[c] [d] [f] [h] [i]	
Passenger Cars and Station Wagons	3-6
Light Trucks	4-6
Heavy Trucks and Truck Tractors	5-9
Off Road Heavy Equipment	6-10

Derived from:

[a] *Capital Stock Measures for Transportation: A Study in Five Volumes*, US Department of Transportation, December 1974.

[b] *Performance of Commercial Activities: Circular A-76 Revised Supplemental Handbook*, OMB March 1996, p.22.
[c] *Performance of Commercial Activities: Circular A-76 Revised Supplemental Handbook*, OMB March 1996, Appendix 3.
[d] *Long Range resource Allocation Plan*, FAA, October 1995.
[e] "A History of Federal Tax Depreciation Policy," Office of Tax Analysis, US Treasury Department, May 1989, Appendix.
[f] *Publication 534*, Internal Revenue Service, November 1995.
[g] *Airport Pavement Design and Evaluation*, FAA Advisory Circular 150/5320-6D, July 7, 1995, p.23.
[h] *Report to the Congress on the Depreciation of Business-Use Passenger Cars*, Department of the Treasury, April 1991.
[i] *Report to the Congress on the Depreciation of Business-Use Light Trucks*, Department of the Treasury, September 1991.

Analysis of regulations which mandate provision of a good or service but which do not specify the method of production are known as performance regulations. They cannot be evaluated over the economic life of the required investments because the equipment has not been specified and its life is, thus, unknown. Performance regulations should be evaluated over the requirement life. The length of time for which a regulation is required must be determined on a case by case basis. In those cases where it is anticipated that the mandated new good or service will become a permanent part of the NAS, the requirement life may be treated as infinite.

Regardless of the evaluation period selected, it should extend over the same number of years for each alternative. This is necessary because benefits and costs are flows and must be measured with respect to time. In certain situations, it will not be possible to compare alternatives with the same number of time periods. This situation frequently arises when an existing facility is being compared with replacements. The existing facility will continue to be functional for sometime; however, its physical life probably will not extend beyond the economic life of the new replacement alternatives. Techniques for dealing with this type of situation are presented in Section III. C.

E. Calculation Issues

1. Basic Procedure

To compute *NPV*, each element of the summation of (5-6) must be evaluated. The first step is to estimate the value of activity benefits each year for each alternative. Next, the cost for each alternative must be estimated and subtracted from the benefit estimates. (Procedures for estimating benefits and costs are developed in Chapters 3 and 4, respectively.) The resulting net benefit in each period t must then be discounted--divided by $(l + r)^t$ --and the resulting values added up to obtain the net present value of the alternative. Such calculations are readily accomplished using a financial calculator or a personal computer. Many calculators and most spreadsheet software, including Excel© and Lotus 1-2-3©, have functions that make this calculation automatically. Should a financial calculator or computer not be available, the computation can be done manually

with the aid of discount factors found in published tables. (Relevant portions of such tables are reproduced in Appendix B.) Table 5-2 presents an example the calculation of these factors for benefits or costs flowing at the end of each period.

TABLE 5-2

REPRESENTATIVE END of PERIOD DISCOUNT FACTORS for 7 PERCENT DISCOUNT RATE

Years from Present	Factor
0	$1/(1+.07)^0 = 1.000$
1	$1/(1+.07)^1 = .935$
2	$1/(1+.07)^2 = .873$
3	$1/(1+.07)^3 = .816$
4	$1/(1+.07)^4 = .763$
5	$1/(1+.07)^5 = .713$

2. Timing of Benefits and Costs

Using the factors in Table 5-2 to discount both benefits and costs occurring in each period would implicitly assume that all benefits and costs occur at the end of a period and are discounted for this period to reflect receipt at the period's end. Actually, several more realistic assumptions are commonly employed with respect to when benefits or costs occur within each period. The most conservative assumption--yielding the lowest NPV for given streams of benefits and costs--is to assume that all costs occur at the beginning of a period and all benefits at the end. The assumption involves discounting the stream of benefits by one more time period than the stream of costs. That is, costs, incurred in the first time period are not discounted at all while benefits in this period are discounted by one period; in the second period, costs are discounted by one period and benefits by two periods, and so on. This assumption is commonly used with financial calculations where money is advanced at the beginning of a period and paid back at the end of the period with interest.

Another common assumption is to assume that all benefits and costs occur at the mid-point of a period. Such a procedure attempts to approximate the reality that benefits and costs occur throughout each period for most investment activities. The discounting procedure involves applying the discount factor for half a period in the first period, one and a half periods in the second period, and so on. Table 5-3 presents an example of such factors. In practice such factors need not be used. All that is necessary is to discount

using end of year factors and then multiply the results by 1.034408. Multiplication by this factor, equal to $(1+.07)^{1/2}$, has the effect of moving all the end of year discounted values closer to the present by a half a year.

TABLE 5-3

REPRESENTATIVE MID-PERIOD DISCOUNT FACTORS for 7 PERCENT DISCOUNT RATE

Years from Present	Factor
0	$1/(1+.07)^{0} = 1.000$
1	$1/(1+.07)^{1/2} = .967$
2	$1/(1+.07)^{1\ 1/2} = .903$
3	$1/(1+.07)^{2\ 1/2} = .844$
4	$1/(1+.07)^{3\ 1/2} = .789$
5	$1/(1+.07)^{4\ 1/2} = .738$

The final assumption commonly employed is that benefits and costs occur continuously over the period and are discounted continuously over the period. This procedure explicitly recognizes that benefits and recurring costs very likely occur throughout a period, rather than at its beginning or end. Moreover, one-time costs projected to occur in the more distant years of an activity's life, such as major overhauls or modifications, are unlikely to occur only on anniversary dates. The continuous procedure assumes an equal probability of the occurrence of such one-time costs throughout the year. Representative discount factors are presented in Table 5-4; complete tables are contained in Appendix B. The computation of these factors is beyond the scope of this guide. The interested reader is referred to any standard engineering economics text.[11]

[11] For example: E. Paul DeGarno and John R. Canada, *Engineering Economy*, Fifth Edition, MacMillan Company, New York, 1973, pp. 143-146.

TABLE 5-4

REPRESENTATIVE CONTINUOUS DISCOUNT FACTORS for 7 PERCENT DISCOUNT RATE

Flow Period	Factor
0 to 1	.967
1 to 2	.904
2 to 3	.845
3 to 4	.789
4 to 5	.738
5 to 6	.689

From a practical point of view, the mid-point and continuous procedures are about the same. Either can be used to approximate the continuous characteristic of benefit and cost streams. Also, there is not a large difference between the end of period discounting and either mid-period or continuous discounting--slightly less than 3.5 percent at a 7 percent discount rate. And assuming costs to occur at the beginning of the period and benefits at the end has the effect of increasing costs relative to benefits by 7 percent. The relatively small changes produced by changing discounting procedures suggests that, with respect to project and regulation evaluation, any of the methods is acceptable. However, the mid-point or continuous procedures have conceptual appeal because they explicitly recognize the continuous nature of benefits and costs. It is recommended that one of these methods be utilized.

3. Special Cases

The computation procedures for determining *NPV* can be simplified substantially in two special situations. The first is where the flow of benefits and costs each period are equal and occur for a finite number of periods. In such cases, the present value of the streams is given by:

$$NPV = \sum_{t=0}^{k} F_t (B - C) \qquad (5\text{-}7)$$

where: F_t = the appropriate discount factor at a given interest rate for the period t periods from today, as discussed above in Section III. E. 1. and given in Tables B-1 and B-2 of Appendix B.

Because *(B-C)* is constant across all periods, it may be removed from the summation to yield (5-8):

$$NPV = (B - C)\sum_{t=0}^{k} F_t \qquad (5\text{-}8)$$

Values for $\sum_{t=0}^{k} F_t$ for various discount rates and values of *k* are tabulated in Tables B-3 and B-4 in Appendix B. Given the evaluation period of an activity, *k*, and the discount rate, the analyst need only determine the appropriate value from the table and multiply it by the annual net benefit amount to determine *NPV*.

A second special case occurs when the flow of benefits and costs each period are equal and occur forever. Such a situation is known as a perpetuity. The present value of such a stream can be calculated very easily by dividing the flow per period by the discount rate, as indicated by equation (5-9).

$$NPV = (B\text{-}C)/r \qquad (5\text{-}9)$$

III. Alternative Decision Criteria

In order to answer the economic questions of (1) which objectives should be pursued and (2) how these objectives should be accomplished, it is necessary to adopt a decision criteria which takes the time distribution of benefits and costs into account. Several proposed criteria may be found in the capital budgeting literature. Four discussed here are net present value, the benefit-cost ratio, uniform annual value, and the internal rate of return. Note that OMB has specified that net present value shall be the standard criteria for deciding whether Government programs can be justified on economic principles. However, OMB encourages presentation of other summary measures as supplementary information to net present value.[12]

A. Net Present Value

The present value (NPV) criterion requires that equation (5-6) be evaluated for all investment or regulatory alternatives. The criterion provides that the alternative to be undertaken (1) have a positive NPV and (2) be that one which has the highest NPV of all alternatives. Condition (1) insures that the activity is worth undertaking; that is, it contributes more in benefits than it absorbs in costs. Condition (2) results in the optimum

[12] "OMB Circular A-94" (Revised--October 29,1992) pp. 3-4.

amount of benefits being efficiently produced. The NPV criterion, then, answers both of the economic questions--what to produce and how to produce it.

As an illustration of the application of NPV, consider the following hypothetical example. An airport is being evaluated for the establishment of a windshear detection system. Three alternatives are being considered: Alternative A--Low Level Windshear Avoidance System (LLWAS), Alternative B--Terminal Doppler Weather Radar (TDWR), and Alternative C--an integrated combination of the two systems (TDWR+LLWAS). Table 5-5 presents the present value of benefits, the present value of costs, the net present value, and the benefit-cost ratio (discussed in section III. B below) for each alternative. Present values are computed using the OMB prescribed 7 percent discount rate and a 20 year project life.

TABLE 5-5

ALTERNATIVE WINDSHEAR AVOIDANCE SYSTEMS
PRESENT VALUES of BENEFITS and COSTS
and BENEFIT-COST RATIOS[a/]
(millions of constant dollars)

Alternative	Benefits	Costs	Benefits Minus Costs	Benefits ÷ Costs
A--LLWAS	$5.2925	$1.0	$4.292	5.29
B--TDWR	$8.820	$4.410	$4.410	2.0
C--TDWR+LLWAS	$9.526	$5.540	$3.986	1.72

[a/]Adapted from "Integrated Wind Shear Systems Cost-Benefit and Deployment Study," Martin Marietta Air Traffic Systems, March 1994, and "Establishment Criteria For Integrated Wind Shear Detection Systems," Report FAA-APO-90-13, December 1990. I should be noted that this is strictly an example, developed to illustrate problems involved in selecting between alternatives. Based on benefit-cost analysis, FAA has in practice installed LLWAS, TDWR, or both at selected airports depending on such factors as airport activity, weather, and technological effectiveness of each system at the time it was installed.

In the hypothetical example of Table 5-5, the LLWAS is substantially cheaper than the TDWR but provides a lower level of benefits than does the TDWR. The combined system has the highest level of benefits and also the highest level of costs. The NPV decision criteria--present value of benefits minus present value of costs--indicates that Alternative B is the best to undertake because it provides the greatest surplus of benefits over costs of the three alternatives.

B. Benefit-Cost Ratio

Another investment criterion is the benefit-cost ratio. It is defined as the present value of benefits divided by costs, and is given by equation (5-10).[13]

$$B/C = \frac{\sum_{t=0}^{k} \frac{B_t}{(1+r)^t}}{\sum_{t=0}^{k} \frac{C_t}{(1+r)^t}} \tag{5-10}$$

The ratio indicates the present value of the dollar benefits that will result per present value dollar invested. A proposed activity with a ratio of at least one will return at least as much in benefits as it costs to undertake. This corresponds to having a positive or zero net present value and indicates that an objective is worth undertaking.

For activities which are independent of each other, the benefit-cost ratio criterion provides a correct answer to the first economic question of which objectives should be undertaken. This is indicated by a benefit-cost ratio greater than or equal to one. FAA has routinely employed benefit-cost ratios in its Facility Establishment Criteria to summarize the benefit-cost relationship for establishing or discontinuing such items as airport traffic control towers, precision approach landing systems, airport surveillance radars, etc. at airports. This procedure has been correct because each proposed installation of a particular facility type is independent of the others. In addition, benefit-cost ratios can be used correctly to rank independent projects as to which are most cost-beneficial. Given the usual constraint of a limited budget, projects can be pursued from highest to lowest benefit-cost ratio until the budget is exhausted.

However, when a selection must be made between competing alternatives, it often fails to completely answer the first economic question as well as correctly answer the second question of how to accomplish the desired objectives most effectively. The difficulty arises in choosing between competing alternatives to accomplish a particular objective which are interdependent. Interdependence occurs when the benefits or costs of one alternative depend on whether or not certain other alternatives are also selected. Interdependence will result in mutual exclusivity--when selection of one alternative precludes selection of any of the others. In some cases the mutually exclusive alternatives relate to the scale of the proposed activity. Selecting one size precludes selecting another. In other cases the mutually exclusive alternatives relate to different methods of achieving

[13] Equation (5-10) is written using discrete, end of period discounting. It could also be stated in terms of any of the other discounting conventions discussed above in Section II. E. of this chapter.

the desired objectives. When competing sizes are involved, the benefit-cost ratio cannot fully answer the first economic question because it does not permit selection between alternative projects. When competing methodologies are involved, it cannot identify the best technique and thus cannot answer the second economic question.

The hypothetical example presented in Table 5-5 demonstrates the interdependence/mutual exclusivity problem. LLWAS and TDWR are both targeted at preventing the same set of windshear accidents. The benefits of implementing one will depend on whether or not the other is already installed. This interdependence is why the benefits of Alternative C (TDWR+LLWAS) are less than the sum of the benefits of the two installed alone. The Alternatives are mutually exclusive because picking Alternative C changes the benefits of Alternatives A and B thus logically precluding them. (Note, in some cases mutual exclusivity will be both logical and physical. For example, building a 100 story building on a particular lot physically precludes building a 200 story building on the same lot.)

From the benefit-cost ratios of the hypothetical example it is not clear which alternative should be selected. All have ratios greater than one indicating that all are cost-beneficial. But, Alternative A has the highest ratio suggesting that it produces the most benefits for the dollars invested and that it should be undertaken. Nonetheless, selecting either Alternative A because it has the highest ratio or Alternative C because it has a positive ratio would be incorrect. To demonstrate that Alternative C is inferior to Alternative B, it is necessary to restructure the data in Table 5-5 to show the incremental benefits and costs of adding TDWR assuming LLWAS is already installed and to show the incremental benefits and costs of adding LLWAS assuming that TDWR is already installed. This information is presented in Table 5-6. As can be seen, adding the second system to the first one has a negative net present value and a benefit-cost ratio less than 1.0 indicating that the incremental addition of the second system is not cost-beneficial.

Having eliminated Alternative C, it remains to choose between Alternative A and Alternative B, which remain mutually exclusive. If the alternative with the highest benefit-cost ratio is selected, it will provide an opportunity to earn the greatest return on the resources actually invested. But, selecting it will preclude earning a positive, albeit smaller return, on additional resources that might be invested under Alternative B. Accordingly, Alternative B--the one with the greatest net present value--is the correct choice. (With mutually exclusive alternatives, only if all the alternatives have the same present value of costs will selecting the benefit-cost ratio with the highest value produce the economically correct result.)

TABLE 5-6

ALTERNATE WINDSHEAR AVOIDANCE SYSTEMS
INCREMENTAL PRESENT VALUES of BENEFITS and COSTS
and INCREMENTAL BENEFIT-COST RATIOS
(millions of constant dollars)

Alternative	Benefits	Costs	Benefits Minus Costs	Benefits ÷ Costs
A--LLWAS	$5.2925	$1.0	$4.292	5.29
C--LLWAS+TDWR	$4.300	$4.5400	-$.24	.95
B--TDWR	$8.820	$4.410	$4.410	2.0
C--TDWR+LLWAS	$.71	$4.5400	-$3.8300	.16

C. Uniform Annual Value

As an alternative to net present value, benefit or cost values may be expressed as annual uniform values (UAV). This involves dividing the present value of a stream of benefits or costs by the same factor that was multiplied by a constant valued stream in equation (5-8) to obtain a present value:

$$UAV = \frac{NPV}{\sum_{t=0}^{k} F_t} \tag{5-11}$$

The factors denoted by $\frac{1}{\sum_{t=0}^{k} F_t}$ are known as capital recovery factors. They may be computed by taking the reciprocal of the values contained in Tables B-3 and B-4 of Appendix B.

The uniform annual method will produce answers to the economic questions which are identical to those produced by the NPV method. This follows by virtue of the fact that all the present values computed under the NPV method need be only divided by the same constant to convert the results to a uniform annual basis. Table 5-7 presents the example of windshear detection systems expressed on a uniform annual cost basis. Also reported

are ratios of annual uniform benefits and costs; note that the ratios are identical to those produced by taking the corresponding ratios of present values as reported in Table 5-6.

TABLE 5-7

ALTERNATIVE WINDSHEAR AVOIDANCE SYSTEMS
INCREMENTAL UNIFORM ANNUAL VALUES of BENEFITS and COSTS
and INCREMENTAL BENEFIT-COST RATIOS
(millions of constant dollars)

Alternative	Uniform Annual Benefits	Uniform Annual Costs	Uniform Annual Benefits Minus Costs	Uniform Annual Benefits ÷ Costs
A--LLWAS	$.4995	$.0943	$.4051	5.29
C--LLWAS+TDWR	$.4059	$.4285	-$.0226	.95
B--TDWR	$.8325	$.4162	$.4162	2.0
C--TDWR+LLWAS	$.0670	$.4285	-$.3615	.16

Historically, the UAV method was widely used for many years, particularly by civil engineers. Its widespread use probably had its origin in Wellington's classic work, *The Economic Theory of Railway Location* (1887). Wellington published during a time when most engineers worked for railways during at least part of their career, and he influenced the thinking of the entire engineering profession. Grant, whose well known book on engineering economy was first published in 1930,[14] prefers to use the UAV method when making comparisons. However, contemporary practice is to use NPV instead of uniform annual values. Not only does the NPV method focus attention on the total net benefits to flow from an activity, it also explicitly identifies the present value of all costs of an undertaking. Comprehensive recognition of costs, discussed in Chapter 4, is known as life cycle costing. Although other summary measures are encouraged to be presented, OMB requires reporting of net present value.[15]

A special UAV application is an exception to the general preference for NPV. In those situations where the alternative methods of accomplishing the objective have unequal lives and (1) the cost estimates associated with the lifetime of any particular alternative may be

[14] Eugene L. Grant and W. Grant Ireson, *Principles of Engineering Economy*, The Ronald Press Company, New York 1964.

[15] "OMB Circular A-94" (Revised--October 29, 1992).

repeated in the future for as many lifetimes as required and (2) the period of required services is either indefinitely long or of a length of time equal to a common multiple of the various alternatives, the UAV method can be used to determine which alternative is best. This requires that the difference between uniform annual benefits and costs be computed as indicated in equation (5-11). Where benefits are identical for alternatives, the same result may be obtained by computing only uniform annual costs and selecting the lowest. It should be noted that where the objective requires provision of a service to a specific future date, the UAV method should not be used. Rather, the NPV method should be computed for each alternative over the required time period.

D. Internal Rate of Return

The internal rate of return (IRR) is defined as that discount rate which equates the present value of the stream of expected benefits in excess of cost to zero. In other words, it is the highest discount rate at which the project will not have a negative NPV. To apply the criterion, it is necessary to compute the IRR and then compare it with the OMB prescribed 7 percent discount rate. If the IRR is greater than or equal to 7 percent the project should be undertaken for its NPV is non-negative. If the IRR is less than 7 percent, the project has a negative NPV and should not be undertaken.

While the IRR method is effective in deciding whether or not a project is worth undertaking, it is difficult to utilize in ranking projects and in deciding between competing mutually exclusive alternatives. It is not unusual for rankings established by the IRR method to be inconsistent with those of the NPV criterion. Moreover, it is possible for a project to have more than one IRR. Although the literature on capital budgeting contains solutions to these problems, these are often complicated or difficult to employ in practice and present opportunities for error.[16] As a consequence, although OMB permits reporting of IRR as supplemental information to NPV, it is not recommended that the IRR method be employed in FAA benefit-cost analyses.

[16] See G. David Quirin, *The Capital Expenditure Decision*, Richard D. Irwin, Inc., Homewood Illinois 1967, pp. 46-55, and Jack Hirshleifer, *Investment, Interest, and Capital*, Prentice-Hall, Inc., Englewood Cliffs, N.J., pp. 71-80.